CBRT - 2020
Question Paper Grid

Government of Goa 10 October 2020

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Assistant Professor(Mathematics)

Itemcode : **PM1122**
Q1 : Photosynthesis is a process

(a) reductive and exergonic
(b) reductive and catabolic
(c) reductive, endergonic and catabolic
(d) reductive, endergonic and anabolic
Key: **D**

Itemcode : **PM1123**
Q2 : Organic Substances which, in very small amounts, control growth and development called

(a) vitamins
(b) hormones
(c) enzymes
(d) None of the above
Key: **B**

Itemcode : **PM1124**
Q3 : The Netaji Subhas National Institute of Sports is located at

(a) Bangalore
(b) Kolkata
(c) Darjeeling
(d) Patiala
Key: **D**

Itemcode : **PM1125**
Q4 : The largest lake in India is

(a) Luni lake
(b) Sambhar lake
(c) Wular lake
(d) None of the above
Key: **C**

Itemcode : **PM1126**
Q5 : The gas usually filled in the electric bulb is

(a) nitrogen
(b) hydrogen
(c) carbon dioxide
(d) oxygen
Key: **A**

Itemcode : **PM1127**
Q6 : Permanent hardness of water may be removed by the addition of

(a) sodium carbonate
(b) alum
(c) potassium permanganate
(d) lime
Key: **A**

Itemcode : **PM1128**
Q7 : Corey Anderson who has hit the fastest ODI century in 36 balls is from

(a) England
(b) Australia
(c) West Indies
(d) New Zealand
Key: **D**

Itemcode : **PM1129**
Q8 : Ball pen function on which one of the following principal?

(a) Boyle's law
(b) Gravitational force
(c) Surface tension
(d) Viscosity
Key: **C**

Itemcode : **PM1130**
Q9 : Outer covering of virus made up of protein is

(a) capsid
(b) coat
(c) virion
(d) viriod
Key: **A**

Itemcode : **PM1131**
Q10 : Which among the following is known as "Sairandhri Vanam"?

(a) Silent Valley National Park
(b) Mudumalai National Park
(c) Periyar National Park
(d) Guindy National Park
Key: **A**

Itemcode : **PM1112**
Q11 : Suppose an implication is derived from a hypothesis, and the implication turns out to be true. This fact:

(a) Makes the hypothesis easier to understand.
(b) Tends to confirm the hypothesis.
(c) Proves the hypothesis true.
(d) Sheds light on the hypothesis
Key: **B**

Itemcode : **PM1113**
Q12 : Hypothetical reasoning is used to produce an explanation for the occurrence of a phenomenon when:

(a) The phenomenon is not observable.
(b) The reason for its occurrence is incomprehensible.
(c) The phenomenon is not measurable.
(d) The reason for its occurrence is not immediately observable
Key: **D**

Itemcode : **PM1114**
Q13 : The letters in the first set have a certain relationship. On the basis of this relationship mark the right choice for the second set: BDFH:OMKI::GHIK:?

(a) FHJL
(b) RPNL
(c) LNPR
(d) LJHF
Key: **C**

Itemcode : **PM1115**
Q14 : Find out the wrong number in the sequence, 52,51,48, 43,34, 27,16

(a) 27
(b) 34
(c) 43
(d) 48
Key: **B**

Itemcode : **PM1116**
Q15 : Four sentences or parts of sentences that form a paragraph are given. Identify the sentence(s) or part(s) of sentence(s) that is/are incorrect in terms of grammar and usage. Then, choose the most appropriate option.

Sentences
P. China is currently affording an opportunity to nations in the region
Q. to become a part of a Beijing-contrived "security alliance", holding forth
R. the promise of a new Asian security paradigm, previously embedded
S. as Chinese President Xi Jinping's "Code of Conduct for Asia".

(a) P only
(b) R only
(c) P and R only
(d) Q and S only
Key: **D**

Itemcode : **PM1117**
Q16 : Here are some words translated from an artificial language.
moolokarn means blue sky

wilkospadi means bicycle race
moolowilko means blue bicycle
Which word could mean "racecar"?

(a) wilkozwet
(b) spadiwilko
(c) moolobreil
(d) spadivolo
Key: **D**

Itemcode : **PM1118**

Q17 : Here are some words translated from an artificial language.
lelibroon means yellow hat
plekafroti means flower garden
frotimix means garden salad
Which word could mean "yellow flower"?

(a) lelifroti
(b) lelipleka
(c) plekabroon
(d) frotibroon
Key: **B**

Itemcode : **PM1119**

Q18 : Each question has an underlined word followed by four answer choices. You will choose the word that is a necessary part of the underlined word.
pain

(a) cut
(b) burn
(c) nuisance
(d) hurt
Key: **D**

Itemcode : **PM1120**

Q19 : Each question has an underlined word followed by four answer choices. You will choose the word that is a necessary part of the underlined word.
gala

(a) celebration
(b) tuxedo
(c) appetizer
(d) orator
Key: **A**

Itemcode : **PM1121**

Q20 : Five books on Mathematics M1, M2, M3,M4 and M5 are arranged in a shelf. Book M5 is arranged next to M1 which is kept on the extreme left and book M2 is not kept next to book M5. Book M4 is kept next to M3 but not M2. Which books are arranged adjacent to M3?

(a) M5 and M2
(b) M1 and M4
(c) M5 and M4
(d) M4 and M2
Key: **C**

Passage:
Education, particularly higher education is the obvious but crucially important instrument for nation building. As Confucious has said:

"If you are thinking of one year, plant rice. If you are thinking of a decade, plant trees. If you are thinking of a century, educate the people."

When we set about the task of higher education, we should be absolutely clear in our perception of the goals of education in the specific context of our nation's development. No doubt, one of the important aims of education would be to create the required range and nature of trained manpower assessed to be needed by different sectors of national growth. The entire educational apparatus must be geared progressively to fulfill the requirements of the different phases of our growth in every sector- primary, secondary and tertiary. The aim must be to ensure that our country does not experience either paucity, or a surfeit of trained manpower in any specific segment of our economy. The requirements of our country, as a free, democratic, secular, socialist, nation, aspiring for rapid development, entail a specific recipe of our educational institutions. Today's educational institutions must therefore be developed accordingly and must regulate themselves to give the country the precise nature and quantum of trained manpower as projected by the requirements of our planned economy.

Itemcode : **PM1132**

Q21 : The author has quoted thoughts of Confucious to stress

(a) importance of planting trees for human beings
(b) the worthless efforts in planting rice
(c) the benefits of investing in education
(d) the need for assessing manpower requirement
Key: **D**

Itemcode : **PM1133**

Q22 : Which statement cannot be made on the basis of the passage?

(a) Higher Education should keep in view the requirements of national economy
(b) Higher education has not been employed for nation building
(c) All levels of education have a role to play in nation's growth
(d) In our country we need to have a specially planned educational system
Key: **D**

Itemcode : **PM1134**

Q23 : The writer believes that

(a) There are no problems related to higher education
(b) investment in education is of long range
(c) higher education should be used to assess manpower needs
(d) aims of higher education in India are absolutely clear
Key: **C**

Itemcode : **PM1135**

Q24 : The writer indicates that

(a) higher education did not play any role in national growth
(b) primary education did not play any role in national growth
(c) our nation experiences paucity of trained manpower in many sectors
(d) today's higher education has no precise goals to achieve
Key: **D**

Itemcode : **PM1136**

Q25 : Author has used the word 'apparatus' to indicate the

(a) scientific nature of education
(b) complicated organization demanded by education
(c) readily visible benefits of education
(d) entire equipment of education to perform particular function
Key: **D**

Itemcode : **PM1062**

Q26 : There is no function from [0,1] to $\mathbb{R}$ that is continuous only on

(a) the set of rational numbers.
(b) the set of all irrational numbers.
(c) $\{\frac{1}{n} : n \in \mathbb{N}\} \cup \{0\}$.
(d) $\{0, \frac{1}{2}, 1\}$
Key: **A**

Itemcode : **PM1063**

Q27 : Let $x \in \mathbb{R}$ such that $x > 0$, and for $n \in \mathbb{N}$, $x_n = x^{\frac{1}{n}}$. Then the sequence (x_n)

(a) always converges to 1.
(b) converges to 0 if $x < 1$ and converges to 1 if $x \geq 1$.
(c) converges to 0 if $x < 1$ converges to 1 if $x = 1$ and diverges if $x > 1$.
(d) converges to 1 if $x \leq 1$ and diverges if $x > 1$.
Key: **A**

Itemcode : **PM1064**

Q28 : Let $x_n = 1 + \frac{-1^n(n+1)}{n}$. Then $\liminf_n x_n$

(a) does not exist.
(b) = 0.
(c) = 1.
(d) = 2.
Key: **B**

Itemcode : **PM1065**

Q29 : If A is the set of all subsets of $\mathbb{N}$ and B is the set of all subsets of $\mathbb{R}$, then

(a) A is countable and B is uncountable.
(b) A and B are bijective with $\mathbb{R}$.
(c) A and B are uncountable and are bijective, but not bijective with $\mathbb{R}$.
(d) A and B are uncountable, but they are not bijective.

Key: **D**

Itemcode : **PM1066**

Q30 : Let $f: (0,1) \to \mathbb{R}$ be a function. Then there is a continuous function $g: [0,1] \to \mathbb{R}$ such that $f(t) = g(t)$ for all $t \in (0,1)$ if

(a) f is continuous.
(b) and only if f is uniformly continuous.
(c) f is differentiable.
(d) f is differentiable and f' is continuous.

Key: **B**

Itemcode : **PM1067**

Q31 : For $n \in \mathbb{N}$, define $f_n: [0,\infty) \to \mathbb{R}$ by $f_n(x) = \frac{nx}{1+nx}$. Then f_n converges pointwise to

(a) a continuous function on $[0,\infty)$ that is differentiable on $(0,\infty)$.
(b) a continuous function that is integrable on $[0,\infty)$.
(c) a function that is not continuous but integrable on $[0,\infty)$.
(d) a function that is neither continuous nor integrable on $[0,\infty)$.

Key: **D**

Itemcode : **PM1068**

Q32 : Let $f(x) = 1$ if $x - \pi \in \mathbb{Q}$ and $= \frac{1}{x^2}$ otherwise, and $g(x) = 1$ if $x \in \mathbb{Q}$ and $= \frac{1}{x}$ otherwise. Then on [0,1],

(a) f and g are Lebesgue integrable.
(b) f is Lebesgue integrable, but g is not Lebesgue intgrable.
(c) g is Lebesgue integrable, but f is not Lebesgue integrable.
(d) neither f nor g is Lebesgue intergable.

Key: **B**

Itemcode : **PM1069**

Q33 : The series $\sum_{n=0}^{\infty}\left(\frac{x^2}{(1+x^2)^n}\right)$ converges to

(a) a differentiable function on $\mathbb{R}$.
(b) a continuous function that is not differentiable on $\mathbb{R}$.
(c) a uniformly continuous function on $\mathbb{R}$.
(d) a function that is not continuous on $\mathbb{R}$.

Key: **D**

Itemcode : **PM1070**

Q34 : Let $f(x) = x^2$ if $x \neq \frac{1}{n}$ and $= 0$ if $x = \frac{1}{n}$, $n \in \mathbb{N}$. If $g(t) = \mathcal{R}\int_0^t f(x)\,dx$ where $\Re\int_0^t dt$ denotes the Riemann integral from 0 to t, then g is

(a) not a well-defined function on[0,1].
(b) a well-defined Lebesgue integrable function on [0,1], but not Riemann integrable on [0,1].

(c) a Riemann integrable function on [0,1], but not continuous on [0,1].

(d) a continuous function on [0,1].

Key: **D**

Itemcode : **PM1071**

Q35 : Let $f(x) = \frac{e^{\left(\frac{-1}{x^2}\right)}}{x^2}$ if $x \neq 0$ and $f(0) = 0$. Then f is

(a) not continuous at 0.

(b) continuous at 0, but not differentiable at 0.

(c) differentiable at 0, but the derivative is not differentiable at 0.

(d) infinitely many times differentiable at 0.

Key: **D**

Itemcode : **PM1072**

Q36 : Let V be a vector space of dimension 12. Let S be a subset of V that has 12 vectors. Which of the following is **FALSE**?

(a) If span$(S) = V$, then every subset of S is linearly independent.

(b) If span$(S) \neq V$, then there must exist a linearly independent subset S_1 of V such that $S \supset S_1$ and S_1 is not a basis for V.

(c) If span$(S) \neq V$, then there must exist a linearly independent subset S_1 of V such that $S \subsetneq S_1$.

(d) If the sum of the vectors in S is the zero vector, then Span$(S) \neq V$.

Key: **C**

Itemcode : **PM1073**

Q37 : Consider $W_1 = \{(x,x) : x \in \mathbb{R}\}$ and $W_2 = \{(x,2x) : x \in \mathbb{R}\}$. Then,

(a) both $W_1 \cap W_2$ and $W_1 \cup W_2$ are vector subspaces of $\mathbb{R}^2$.

(b) neither $W_1 \cap W_2$ nor $W_1 \cup W_2$ is a vector subspace of $\mathbb{R}^2$.

(c) $W_1 \cap W_2$ is a vector subspace of $\mathbb{R}^2$ but $W_1 \cup W_2$ is not.

(d) $W_1 \cap W_2$ is not a vector subspace of $\mathbb{R}^2$ but $W_1 \cup W_2$ is.

Key: **C**

Itemcode : **PM1074**

Q38 : If for a real square matrix A, $A^T = A^{-1}$, then A is

(a) normal.

(b) symmetric.

(c) hermitian.

(d) orthogonal.

Key: **D**

Itemcode : **PM1075**

Q39 : Let W be a subspace of $\mathbb{R}^n$ and $W^\perp$ denotes its orthogonal complement. If W_1 is a subspace of $\mathbb{R}^n$ such that for all $x \in W_1$ the dot product $x \cdot u = 0$ for all $u \in W^\perp$. Then,

(a) dim $W_1^\perp \leq$ dim $W^\perp$.

(b) dim $W_1^\perp \leq$ dim W.

(c) dim $W_1^\perp \geq$ dim $W^\perp$.

(d) dim$W_1^\perp \geq$ dim W.

Key: **C**

Itemcode : **PM1076**

Q40 : Let A be a 5×5 matrix with real entries and $x \in \mathbb{R}^5, x \neq 0$. Then the vectors $x, Ax, A^2x, A^3x, A^4x, A^5x$ are

(a) linearly independent.

(b) linearly dependent.

(c) linearly independent if and only if A is symmetric.

(d) linear independent if $\det(A) \neq 0$.

Key: **B**

Itemcode : **PM1077**

Q41 : Which one of the subset of $C[0,1]$ is linearly dependent over $\mathbb{R}$?

(a) $\{1, x, x^2, x^{2x}\}$.

(b) $\{1, e^x, e^{2x}, e^{3x}\}$.

(c) $\{1, \cos x, \cos 2x, (\cos x)^2\}$.

(d) $\{1, \sin x, \sin 2x, (\sin x)^2\}$.

Key: **C**

Itemcode : **PM1078**

Q42 : Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined as $f(x,y) = x$ if $y = 0$, $= y$ if $x = 0$, and $= 0$ otherwise. At $(0,0)$, f

(a) is differentiable.

(b) admits all directional derivatives, but not differentiable.

(c) admits partial derivatives, but not all directional derivatives.

(d) continuous, but does not admit partial derivatives.

Key: **B**

Itemcode : **PM1079**

Q43 : Let $F: \mathbb{R}^2 \to \mathbb{R}^2$ be defined as $F(x,y) = (x^2 + y^2, 2xy)$. Then F

(a) is one-one and maps open sets to open sets.

(b) is onto $\mathbb{R}^2$ mapping open sets to open sets.

(c) is a bijection mapping open sets to open sets.

(d) is a bijection, but is not an open mapping.

Key: **B**

Itemcode : **PM1080**

Q44 : Let U be an open subset of $\mathbb{R}^n$ and $f: U \to \mathbb{R}^n$ be a C^1-map such that $f'(u)$ is one-one at every point $u \in U$. Then,

(a) f is one-one and maps open subsets of U to open subsets of $\mathbb{R}^n$.

(b) f is one-one, but may not map open subsets of U to open subsets of $\mathbb{R}^n$.

(c) f may not be one-one, but maps open subsets of U to open subsets of $\mathbb{R}^n$.

(d) neither f is one-one nor f maps open subsets of U to open subsets of $\mathbb{R}^n$.

Key: **C**

Itemcode : **PM1081**

Q45 : Let $f: \mathbb{R}^5 \to \mathbb{R}^2$ be defined as $f(x,y,u,v,w) = (x^2 + y^2 - u + 2v + w^2, \frac{x^2}{2} + y - u + \frac{v^2}{2} + \frac{w^3}{3})$. Then, in some neighbourhood of $(1,1,1,1,1)$,

(a) x, y and u can be expressed as functions of v and w.

(b) y, u and v can be expressed as functions of w and x.

(c) u, v and w can be expressed as functions of x and y.

(d) v, w and x can be expressed as functions of y and u.

Key: **D**

Itemcode : **PM1082**

Q46 : Let $f(x,y) = x^2 - xy - 2y^2$. Then, at $(0,0)$, f admits

(a) a point of inflection.
(b) a local maximum.
(c) a local minimum.
(d) a regular (non-critical) point.

Key: **A**

Itemcode : **PM1083**

Q47 : The Lebesgue measure of $\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < 1\} \cup \{(x,y) \in \mathbb{R}^2 : y = 0\}$ in $\mathbb{R}^2$ is

(a) 0.
(b) π.
(c) 1.
(d) Infinite.

Key: **B**

Itemcode : **PM1084**

Q48 : The number of elements of order 6 in S_6 is

(a) 120.
(b) 240.
(c) 840.
(d) 1440.

Key: **B**

Itemcode : **PM1085**

Q49 : Let $n > 4$ and H be the smallest normal subgroup of S_n containing the cycle (1,2,3). Then

(a) $H = S_n$.
(b) $H = A_n$.
(c) $3 < o(H) < \frac{n!}{2}$.
(d) $H = \{(e, (1,2,3), (1,3,2)\}$.

Key: **B**

Itemcode : **PM1086**

Q50 : It is true that

(a) there is a simple non-abelian simple group of order 65.
(b) there is a non-simple non-ableian group of order 65.
(c) there is a non-cyclic abelian group of order 65.
(d) every group of order 65 is cyclic.

Key: **D**

Itemcode : **PM1087**

Q51 : The set of all continuous real valued functions $C[0,1]$ on [0,1] under pointwise addition and multiplication is

(a) a field.
(b) an integral domain, but not a field.
(c) a commutative ring, but not an integral domain.
(d) not a commutative ring.

Key: **C**

Itemcode : **PM1088**

Q52 : The number of non-zero ring homomorphisms from $\mathbb{Z}$ to $\mathbb{Z}$ is

(a) 1.

(b) 2.

(c) 3.

(d) Infinite.

Key: **A**

Itemcode : **PM1089**

Q53 : Let I_1 and I_2 be the ideals in $\mathbb{Z}_7[x]$ generated by $x^3 + x + 2$ and $x^2 + 2x + 1$ respctively and the quotient ring $\mathbb{Z}_7[x]/I_k$ be denoted by $F_k,\ k = 1, 2$. Then,

(a) F_1 and F_2 are fields.

(b) F_1 is a field and F_2 is not a field.

(c) F_2 is a field and F_1 is not a field.

(d) neither F_1 nor F_2 is a field.

Key: **D**

Itemcode : **PM1090**

Q54 : Let τ_1, τ_2 and τ_3 be the topologies on $\mathbb{R}$ generated by the subbases $\mathcal{B}_1 = \{(a, b): a, b \in \mathbb{R}, a < b\}, \mathcal{B}_2 = \{[a, b): a, b \in \mathbb{R}, a < b\}$, and $\mathcal{B}_3 = \mathcal{B}_1 \cup \mathcal{B}_2$ respectively. Then

(a) $\tau_1 \subsetneq \tau_2 \subsetneq \tau_3$.

(b) $\tau_1 = \tau_2 \subsetneq \tau_3$.

(c) $\tau_1 \subsetneq \tau_2 = \tau_3$.

(d) $\tau_1 = \tau_2 = \tau_3$.

Key: **C**

Itemcode : **PM1091**

Q55 : Let X be a compact Hausdorff space, Y an open subset of X and Z a closed subset of X. Then,

(a) Y and Z are locally compact.

(b) Y is locally compact, but Z is not.

(c) Y is not locally compact, but Z is.

(d) neither Y nor Z is locally compact.

Key: **A**

Itemcode : **PM1092**

Q56 : Let Y be an open connected subset of a locally path connected space X. Then

(a) Y is locally path connected, but need not be path connected.

(b) Y is path connected, but need not be locally path connected.

(c) Y is both path connected and locally path connected.

(d) Y is neither locally path connected nor path connected.

Key: **C**

Itemcode : **PM1093**

Q57 : Let $d_1(x, y) = |x - y|$ and $d_2(x, y) = |\frac{1}{x} - \frac{1}{y}|$, and X_1 and X_2 be (0,1] with the metrics d_1 and d_2 respectively. Then,

(a) X_1 is a complete metric space, but X_2 is not a complete metric space.

(b) X_1 is not a complete metric space, but X_2 is a complete metric space.

(c) X_1 and X_2 are complete metric spaces.
(d) neither X_1 nor X_2 is a complete metric space.
Key: **B**

Itemcode : **PM1094**
Q58 : $\mathbb{R}_l$ ($\mathbb{R}$ with the lower limit topology) is

(a) first and second countable and separable.
(b) first and second countable, but not separable.
(c) first countable and separable, but not second countable.
(d) first countable, but neither second countable nor separable.
Key: **C**

Itemcode : **PM1095**
Q59 : Let $f(z)$ be an analytic function on $\mathbb{C}$. Then,

(a) $f(\bar{z})$ is analytic on $\mathbb{C}$.
(b) $\overline{f(z)}$ is analytic on $\mathbb{C}$.
(c) $\overline{f(\bar{z})}$ is analytic on $\mathbb{C}$.
(d) $|f(\bar{z})|$ is analytic on $\mathbb{C}$.
Key: **C**

Itemcode : **PM1096**
Q60 : There is a non-constant analytic function from

(a) $\mathbb{C}$ **onto** $\{z \in \mathbb{C}: |z| < 1\}$.
(b) $\{z \in \mathbb{C}: |z| < 1\}$ **onto** $\mathbb{R}$.
(c) $\mathbb{C}$ **onto** $\{z \in \mathbb{C}: |z| = 1\}$.
(d) $\{z \in \mathbb{C}: |z| < 1\}$ **onto** $\{z \in \mathbb{C}: Re(z) > 0\}$.
Key: **D**

Itemcode : **PM1097**
Q61 : Let D be a domain in $\mathbb{C}$, $f: D \to \mathbb{C}$ be complex differentiable on D and $z \in D$. Then

(a) f admits a series expansion in a neighbourhood of z.
(b) f is infinitely many times complex differentiable, but need not have a series expansion in any neighbourhood of z.
(c) f need not be infinitely many times differentiable at z, but f' is continuous at z.
(d) f' need not be continuous.
Key: **A**

Itemcode : **PM1098**
Q62 : There is a non-constant analytic function f: $\{z \in \mathbb{C}: |z| < 1\} \to \mathbb{C}$ such that

(a) $|f(z)| = 1$ for all z such that $0 < |z| < \frac{1}{2}$.
(b) $f(z) = 0$ for infinitely many points z such that $0 < |z| < \frac{1}{2}$.
(c) $f(z) \in \mathbb{R}$ for all z such that $0 < |z| < \frac{1}{2}$.
(d) $|f(z)| < |z|$ for all z such that $0 < |z| < \frac{1}{2}$.
Key: **D**

Itemcode : **PM1099**
Q63 : $f(z) = \frac{1}{\sin z}$ and $g(z) = \sin\frac{1}{z}$. Then,

(a) f and g admit essential singularity at 0.

(b) f admits an essential singularity at 0 and g admits a pole at 0.

(c) f admits a pole at 0 and g admits an essential singularity at (

(d) both f and g admit a pole at 0.

Key: **C**

Itemcode : **PM1100**

Q64 : Let γ be the positively oriented curve $|z| = 2$. Then $\oint_\gamma \frac{dz}{z^2+2} =$

(a) 0.

(b) $\frac{1}{4\pi i}$.

(c) $\frac{1}{8\pi i}$.

(d) $\frac{-1}{4\pi i}$.

Key: **A**

Itemcode : **PM1101**

Q65 : Consider the norms $\|x\|_1 = \max_{t\in[0,1]}|x(t)|$ and $\|x\|_2 = \int_0^1 |x(t)|\,dt$ on the real vector space $X = C[0,1]$ of all real valued continuous functions on [0,1]. Then,

(a) $\| \quad \|_1$ is given by an inner product on X, but $\| \quad \|_2$ is not given by any inner product on X.

(b) $\| \quad \|_2$ is given by an inner product on X, but $\| \quad \|_1$ is not given by any inner product on X.

(c) both the norms are obtained from inner products on X.

(d) neither of the norms is obtained from any inner product on X.

Key: **D**

Itemcode : **PM1102**

Q66 : Let X and Y be normed linear spaces and $T: X \to Y$ be a bounded linear operator. Which of the following statements is **not** true?

(a) T is continuous.

(b) T maps every bounded subset of X to bounded subset of Y.

(c) T maps unbounded subsets of X onto unbounded subset of Y.

(d) T maps Cauchy sequences in X to a Cauchy sequences in Y.

Key: **C**

Itemcode : **PM1103**

Q67 : The norm of the linear functional f defined on $C[-1,1]$ by $f(x) = \int_{-1}^0 x(t)dt - \int_0^1 x(t)dt$ with respect to the sup norm on $C[-1,1]$ is

(a) 0.

(b) 1.

(c) 2.

(d) not finite.

Key: **C**

Itemcode : **PM1104**

Q68 : For a non-empty subset A of an inner product space X,

(a) $A^{\perp\perp} \subset A$,

(b) $A^{\perp\perp} \supset A$.

(c) $A^{\perp\perp} = A$.

(d) $A^{\perp\perp\perp} = A$.

Key: **B**

Itemcode : **PM1105**

Q69 : Let X be a normed linear space and X' its dual space. Which of the following statements is **not** true?

(a) If X is separable, then X' is separable.

(b) If X' is separable, then X is separable.

(c) If X is reflexive, then X' is reflexive.

(d) If X'' is reflexive, then X' is reflexive.

Key: **A**

Itemcode : **PM1106**

Q70 : Let $G(t,s) = \begin{cases} g_1(t,s) & \text{if } t \le s \\ g_2(t,s) & \text{if } s \le t \end{cases}$ be a Green's function for the equation

$x''(t) = f(t);\ x(0) = 0 = x(1)$. Then, $g_1(t,s) =$

(a) $s(1-t)$.

(b) $t(1-s)$.

(c) $-t(s+1)$.

(d) $-s(t+1)$.

Key: **B**

Itemcode : **PM1107**

Q71 : Every solution ϕ of the equation $y'' + \omega^2 y = A\cos\ \omega x$, where A and ω are positive constants, is such that as $x \to \infty$, $|\phi(x)|$

(a) takes arbitrary large values.

(b) takes a non-zero finite value.

(c) tends to 0.

(d) none of these.

Key: **A**

Itemcode : **PM1108**

Q72 : Solution to the equation $xy'' + (1-x)y' + ny = 0$, n is a non-negative integer, is given by $y(x) =$

(a) $e^{-x}\frac{d^n}{dx^n}(x^n e^x)$.

(b) $e^{x}\frac{d^n}{dx^n}(x^n e^x)$.

(c) $e^{-x}\frac{d^n}{dx^n}(x^n e^{-x})$.

(d) $e^{x}\frac{d^n}{dx^n}(x^n e^{-x})$.

Key: **D**

Itemcode : **PM1109**

Q73 : Suppose $u(x,y)$ is a non-constant function harmonic in a bounded domain D and continuous on $\overline{D} = D \cup B$ where B is the boundary of D. Then u attains

(a) its maximum on D.

(b) its minimum on D.

(c) its maximum on B.

(d) neither maximum nor minimum on $\overline{D}$.

Key: **C**

Itemcode : **PM1110**

Q74 : The characteristic curves of the equation $xz_y + yz_x = z$ are given by

(a) $z = k(x^2 + y^2)e^{-\sin^{-1}(\frac{x}{e})}$.

(b) $x^2 + y^2 = c$.

(c) $x^2 - y^2 = c$.

(d) $z = k\sqrt{(x^2 + y^2)}e^{-\sin^{-1}(\frac{x}{e})}$.

Key: **B**

Itemcode : **PM1111**

Q75 : The Riemann function for the wave equation in the canonical form $u_{xy} = 0$ is given by $v(x, y;\ \alpha, \beta) =$

(a) $J_0(\sqrt{(x-\alpha)(y-\beta)})$.

(b) $\frac{(x+y)(\alpha-\beta)}{(\alpha+\beta)^2}$.

(c) $J_0(\sqrt{y})$.

(d) 1.

Key: **D**